Extracting maximum information from limited data

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Outline

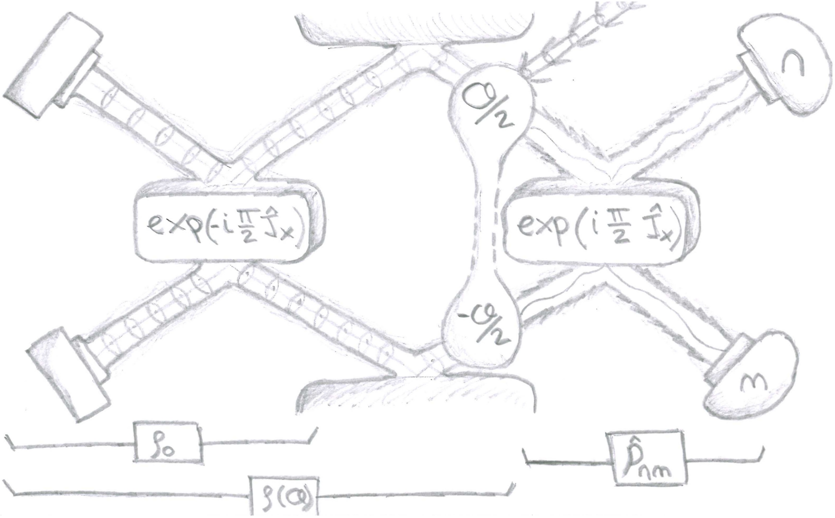
Quantum metrology protocols

Enunciation of the problem

Mach-Zehnder interferometry with limited data and moderate prior information

Conclusions

Quantum metrology protocols



Probe preparation

$$\text{Experimental arrangement} \longrightarrow \rho_0$$

Unknown parameter encoding

$$\rho_0 \longrightarrow \rho(\theta) = U(\theta)\rho_0 U^\dagger(\theta)$$

Measurement scheme and data read-out

$$E(n) \longrightarrow \text{outcome } n,$$

$$\text{with probability } p(n|\theta) = \mathrm{Tr}\left[E(n)\rho(\theta)\right]$$

Parameter information summary

$$\text{prior } p(\theta),\ \text{likelihood } p(n|\theta) \longrightarrow p(\theta, n) = p(\theta)p(n|\theta)$$

Parameter estimation

$$p(\theta, n) \longrightarrow \begin{cases} \text{estimate} : g(n) \\ \text{measure of uncertainty} : \sqrt{\bar{\epsilon}} \end{cases}$$

[1] R. Demkowicz-Dobrzański et al., Progress in Optics, **60**, 345 (2015).

[2] E. T. Jaynes, Cambridge University Press (2003).

Enunciation of the problem

Motivation

Given μ observations $\boldsymbol{n} = (n_1, n_2, \ldots, n_\mu)$, if $\mu \gg 1 \longrightarrow \bar{\epsilon} \approx 1/(\mu F)$, where

$$F = \int_\Delta \frac{dn}{p(n|\theta)} \left[\frac{\partial p(n|\theta)}{\partial \theta}\right]^2 \Bigg|_{\theta=0}$$

However,

- if we want to study fragile systems, or
- the system under study is out of reach after a few observations,

then the previous formalism is not suitable.

[3] Jesús Rubio, Paul Knott and Jacob Dunningham, J. Phys. Comm., 2(1):015027 (2018).

Some strategies

- Bayesian quantum bounds (Ziv-Zakai, Weiss-Weinstein, optimal bias):

  [4] M. Tsang, Phys. Rev. Lett., **108**, 230401 (2012).

  [5] X.-M- Lu and M. Tsang, Quantum Science and Technology, 1(1):015002 (2016)

  [6] J. Liu and H. Yuan, New Journal of Physics, 18(9):093009 (2016)

- Direct numerical approaches (e.g., using Monte Carlo simulations or machine learning):

  [7] S. L. Braunstein et al, Phys. Rev. Lett., 69:2153-2156 (1992)

  [8] A. Lumino et al, arXiv: 1712.07570 (2017)

- *Non-asymptotic analysis of those protocols that have been optimised using the asymptotic theory:*

  [3] Jesús Rubio, Paul Knott and Jacob Dunningham, J. Phys. Comm., 2(1):015027 (2018).

Our strategy

1. Best estimation scheme for a single shot, with

    a) fixed ρ_0,

    b) fixed $U(\theta)$,

    c) and a flat prior for $\theta \in [-W/2, W/2]$.

2. μ repetitions of the optimal single-shot strategy.

3. Study of the uncertainty as a function of μ.

Mach-Zehnder interferometry with limited data and moderate prior information

Single-shot measure of uncertainty

$$\bar{\epsilon} = \int_{\Delta} dn \; d\theta \; p(\theta) \mathrm{Tr}\left[E(n)\rho(\theta)\right] \, 4 \, \sin^2 \left[\frac{g(n) - \theta}{2}\right]$$

If the width W satisfies that $W \lesssim 2$, then

$$\bar{\epsilon} \approx \bar{\epsilon}_{\mathrm{mse}} = \int_{\Delta} dn \; d\theta \; p(\theta) \mathrm{Tr}\left[E(n)\rho(\theta)\right] [g(n) - \theta]^2$$

[1] R. Demkowicz-Dobrzański et al., Progress in Optics, **60**, 345 (2015).

[3] Jesús Rubio, Paul Knott and Jacob Dunningham, J. Phys. Comm., 2(1):015027 (2018).

Optimal POVM and estimator for a single shot

$$\bar{\epsilon} \approx \bar{\epsilon}_{\mathrm{mse}} \geqslant \int_a^b d\theta p(\theta)\theta^2 - \mathrm{Tr}\left(S\bar{\rho}\right),$$

where

$$\rho = \int_a^b d\theta p(\theta)\rho(\theta), \quad \bar{\rho} = \int_a^b d\theta p(\theta)\rho(\theta)\theta \quad \text{and} \quad S\rho + \rho S = 2\bar{\rho}.$$

The optimal strategy is then given by

$$S = \int_\Delta ds\ s \left|s\middle\rangle\middle\langle s\right|,$$

where s are the estimates and $E(s) = |s\rangle\langle s|$ are the POVM elements.

[9] S. Personick, IEEE, 17(3):240-246 (1971).

[10] K. Macieszczak et al., New Journal of Physics **16**, 113002 (2014).

Measure of uncertainty for μ observations

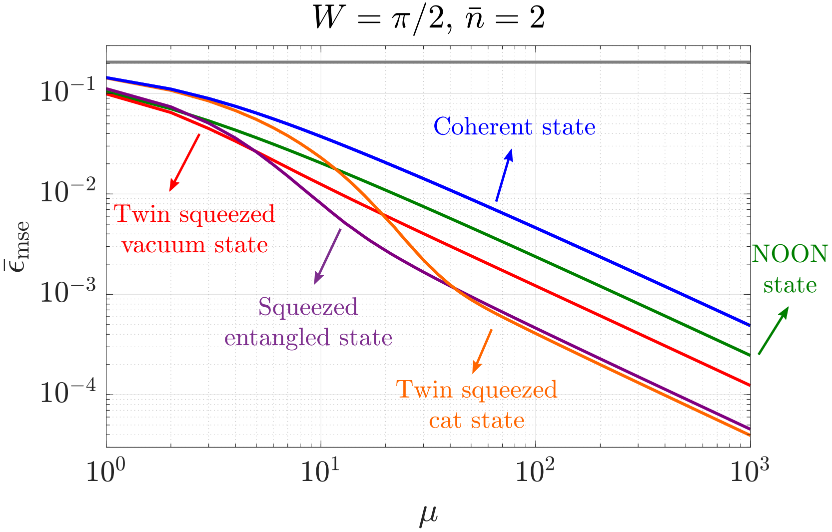
$$\bar{\epsilon} \approx \bar{\epsilon}_{\mathrm{mse}} = \int_{\Delta} d\boldsymbol{s} d\theta p(\theta) p(s_1|\theta) \dots p(s_\mu|\theta) [g(\boldsymbol{s}) - \theta]^2$$

$$= \int_{\Delta} d\boldsymbol{s} d\theta p(\theta) \mathrm{Tr}\left[E(s_1)\rho(\theta)\right] \dots \mathrm{Tr}\left[E(s_\mu)\rho(\theta)\right] [g(\boldsymbol{s}) - \theta]^2,$$

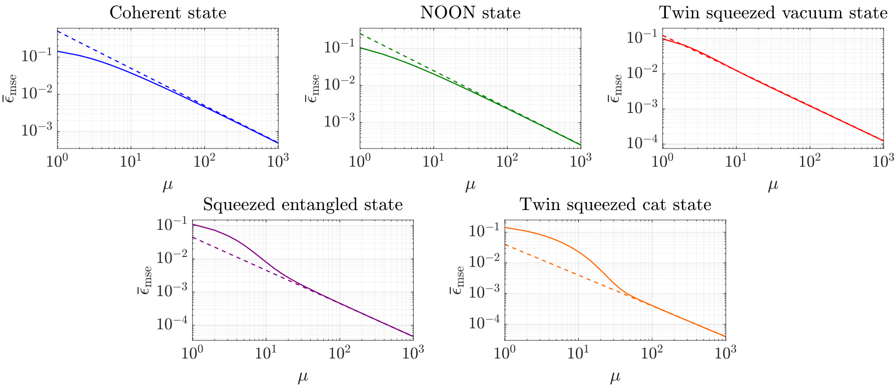
where $\boldsymbol{s} = (s_1, s_2, ..., s_\mu)$.

[3] Jesús Rubio, Paul Knott and Jacob Dunningham, J. Phys. Comm., 2(1):015027 (2018).

Optical quantum metrology

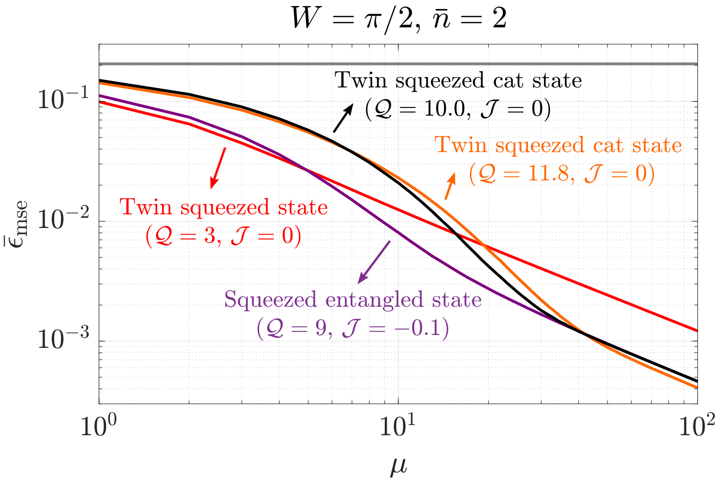


Approaching the quantum Cramér-Rao bound



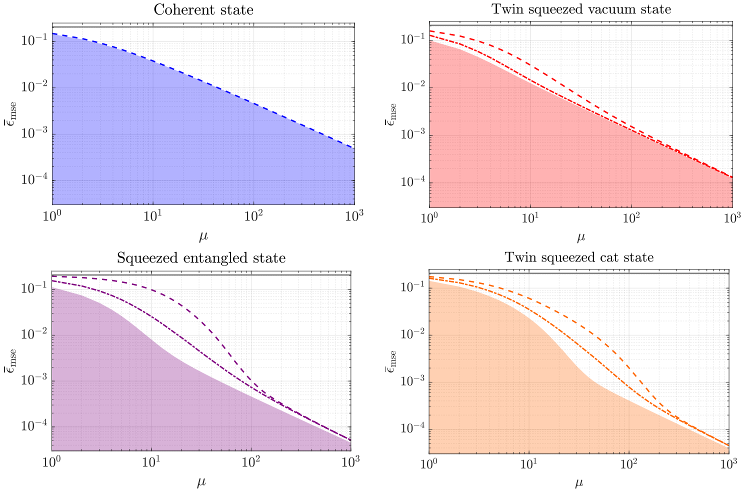
[3] Jesús Rubio, Paul Knott and Jacob Dunningham, J. Phys. Comm., 2(1):015027 (2018).

The role of quantum correlations



[11] J. Sahota and N. Quesada, Phys. Rev. A, 91:013808 (2015).

Practical measurements



[3] Jesús Rubio, Paul Knott and Jacob Dunningham, J. Phys. Commun., 2(1):015027 (2018).

Conclusions

- A methodology for quantum metrology of experiments that operate in the regime of low μ has been developed.
- We have recovered the results of the local theory in the asymptotic regime.
- We have discussed the role of quantum correlations for low μ and the possibility of approaching our bounds with practical measurements.

A preliminary framework for these ideas is available in

[3] Jesús Rubio, Paul Knott and Jacob Dunningham, *Non-asymptotic analysis of quantum metrology protocols beyond the Cramér-Rao bound*, **J. Phys. Comm., 2(1):015027 (2018)**

and a more complete version including the results of this talk will appear shortly on the arXiv. Stay tuned!

Thank you for your attention

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South East Physics Network

Networked Quantum Information Technologies

UK National Quantum Technologies Programme

Work developed with Jacob Dunningham (supervisor)
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