Quantum thermometry with adaptive Bayesian strategies: a case study for release-recapture experiments

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Key works:

arXiv:2204.11816 (accepted in PRX Quantum)
Quantum Sci. Technol. (8) 015009, 2022
Phys. Rev. Lett. (127) 190402, 2021

with

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JSPS London Symposium, Nottingham

15th Dec 2022

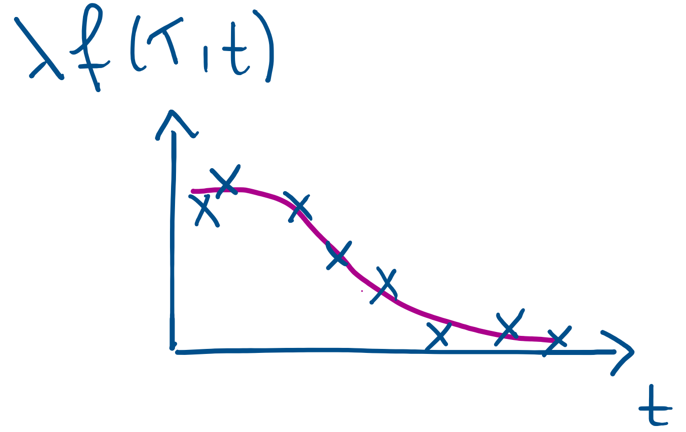
Our plan for today

I. Release-recapture thermometry

II. Bayes meets thermometry: a quick journey through the foundations of scale estimation

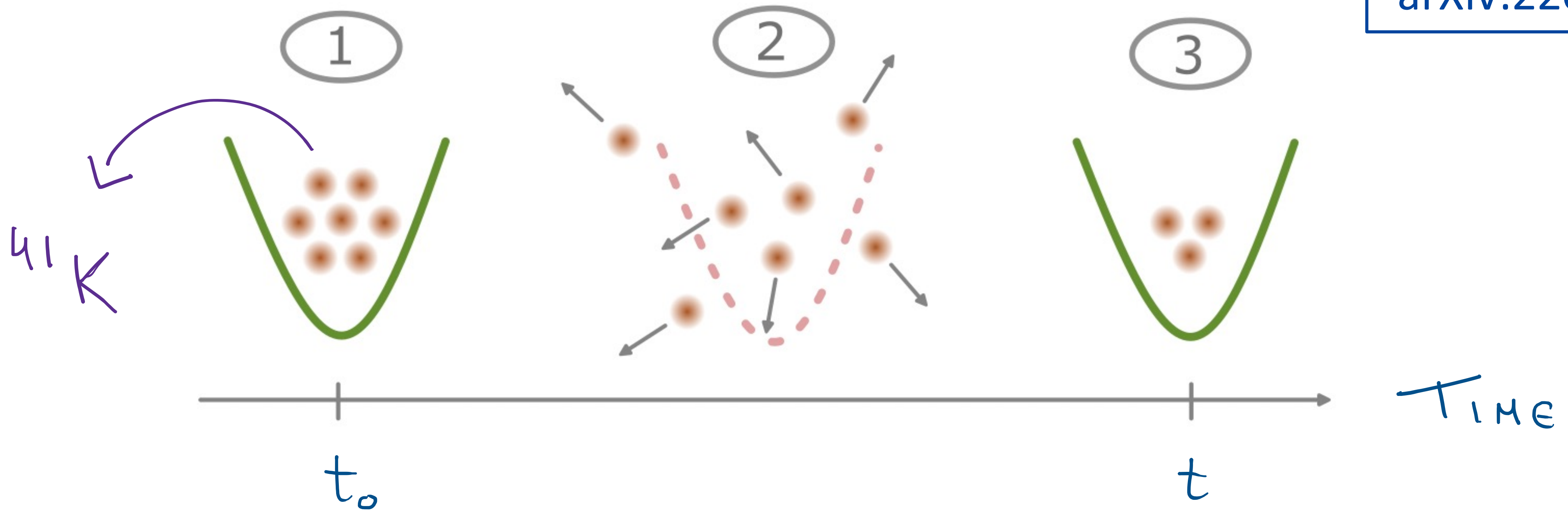
III. Maximising information content in an adaptive fashion

IV. Conclusions and outlook



I. Release-recapture thermometry: an overview

arXiv:2204.11816



1) N_0 atoms at temperature T are trapped

2) Trap switched off $\Rightarrow$ atoms expand ballistically

3) $n \leq N_0$ atoms are recaptured

I. Release-recapture thermometry: an overview

After M trials :

$$\left.\begin{aligned}\vec{n} &:= (n_1, n_2, \dots, n_M) \\ \vec{t} &:= (t_1, t_2, \dots, t_M)\end{aligned}\right\} \text{data}$$

numbers of recaptured atoms

expansion times ≡ controlled parameter

I. Release-recapture thermometry: an overview

standard approach

PROTOCOLS FOR TEMPERATURE ESTIMATION

(i) Least squares

(ii) Unoptimised Bayes

(iii) A priori optimised

(iv) Fully adaptive

global quantum thermometry

Phys. Rev. Lett. **127**, 190402 (2021)

Quantum Sci. Technol. **8** 015009 (2022)

Least squares (standard approach)

$$\vec{n} = (\underbrace{n_{1,1}, \dots, n_{1 d_1}}, \dots, \underbrace{n_{\nu 1}, \dots, n_{\nu d_\nu}})$$

$$\left(\underbrace{\sum_{i=1}^{\nu} d_i = M}_{} \ \text{trials} \right)$$

$$\left.\boxed{\begin{aligned} \langle \vec{n} \rangle &= (\langle n_1 \rangle, \dots, \langle n_\nu \rangle) \\ \vec{t} &= (t_1, \dots, t_\nu) \end{aligned}}\ \right\} \text{data}$$

Least squares (standard approach)

$$\langle \vec{n} \rangle = (\langle n_1 \rangle, \ldots, \langle n_\nu \rangle)$$
$$\vec{t} = (t_1, \ldots, t_\nu)$$

data

fraction of recaptured atoms

unknown temperature

From statistical mechanics:

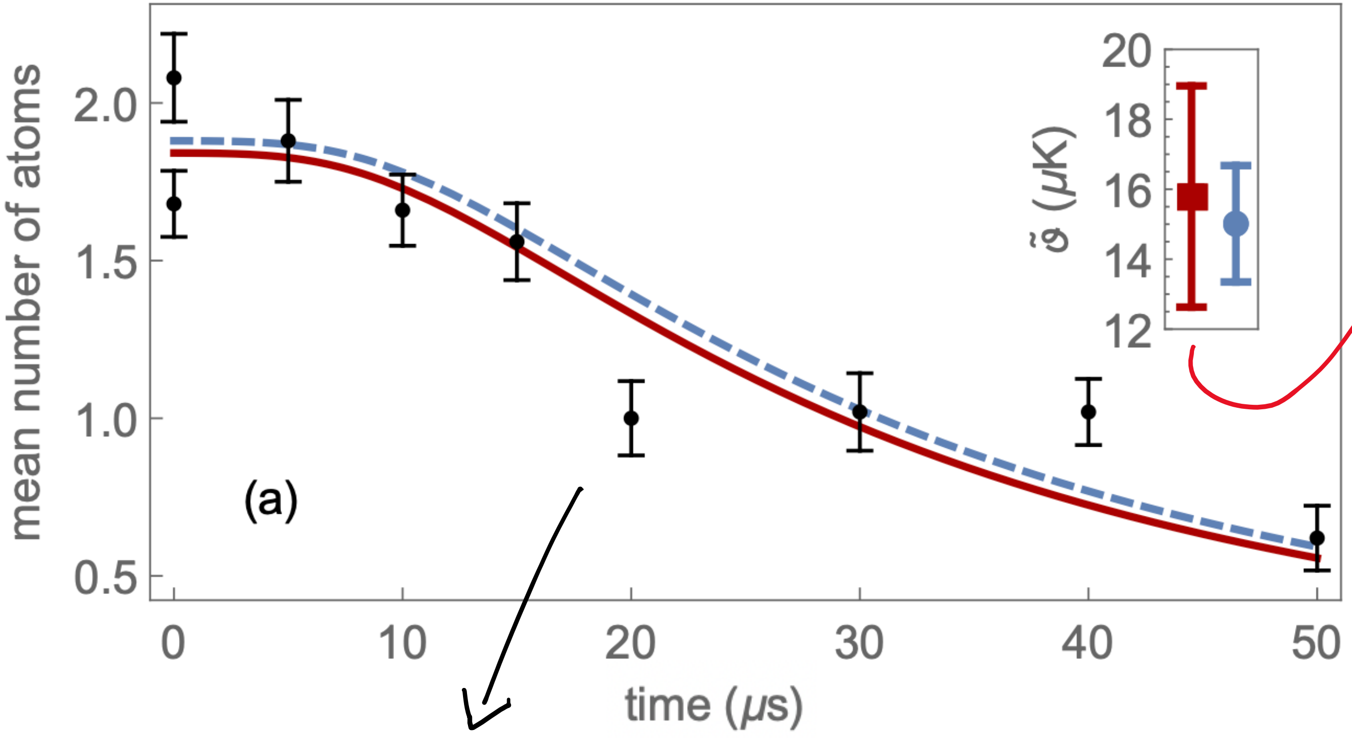
$$\langle n(t) \rangle \quad \longleftrightarrow \quad \lambda \, f(T, t)$$

mean number of atoms

fitting parameters

Least squares (standard approach)

Fitting to $\langle n(t) \rangle \overset{\sim}{\longleftrightarrow} \lambda f(T, t)$, we get:

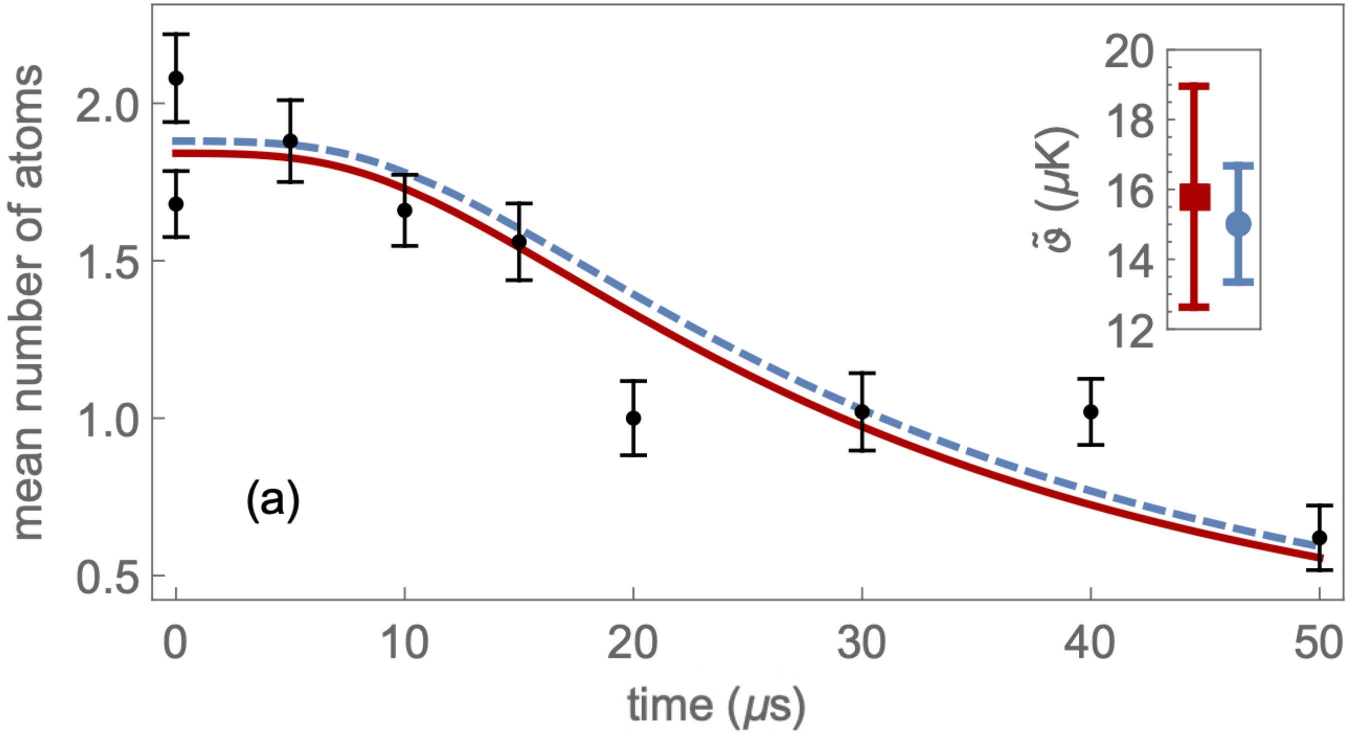


$\langle n_i \rangle$ at time t_i

$\tilde{T} = 15.8 \pm 3.2\ \mu K$

FINAL TEMPERATURE ESTIMATE

Least squares (standard approach)

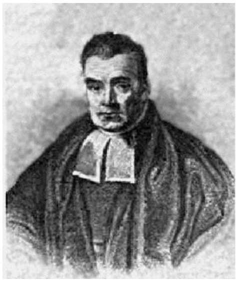


$T = 15.8 \pm 3.2\ \mu K$

- Why another method?
  - → Fails to use all available info
  - → Wastes resources
- With global quantum thermometry (this talk):

Twice as much precision with half of the measurement data

II. Bayes meets thermometry: a quick journey through the foundations of scale estimation



$\boxed{1}$ Construct a deviation function D:

hypothesis

estimator

$$D[\tilde{\Theta}_{\vec{y}}(\vec{m}), \Theta]$$

measurement record

control parameters

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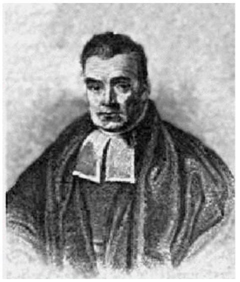
II. Bayes meets thermometry: a quick journey through the foundations of scale estimation



$\boxed{1}$ Construct a deviation function $D[\tilde{\Theta}_{\vec{y}}(\vec{m}), \Theta]$

$\boxed{2}$ Construct an error functional $\bar{\epsilon}_{\vec{y}}$:

$$\bar{\epsilon}_{\vec{y}} = \int d\Theta \, d\vec{m} \, \underbrace{p(\Theta, \vec{m} | \vec{y})}_{\text{joint probability (physical assumptions)}} D[\tilde{\Theta}_{\vec{y}}(\vec{m}), \Theta]$$

parameter independent

measurement independent

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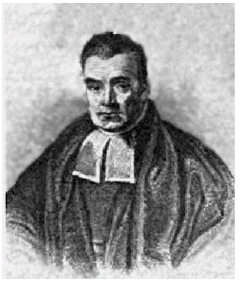
II. Bayes meets thermometry: a quick journey through the foundations of scale estimation



1 Construct a deviation function $D[\tilde{\theta}_{\vec{y}}(\vec{m}), \theta]$

2 Construct an error functional

$$\bar{\epsilon}_{\vec{y}} = \int d\theta \, d\vec{m} \, p(\theta, \vec{m} | \vec{y}) \, D[\tilde{\theta}_{\vec{y}}(\vec{m}), \theta]$$

3 Minimise $\bar{\epsilon}_{\vec{y}}$ over:

- Estimator $\tilde{\theta}_{\vec{y}}(\vec{m})$
- Control parameters $\vec{y}$
- POVM $\Pi_{\vec{y}}(\vec{m})$

optimisation problem

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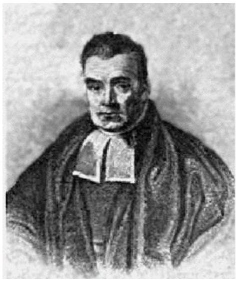
Release-recapture thermometry: unoptimised times



- Physical assumptions:

likelihood

$$p(\omega, \vec{n} \mid \vec{t}) = p(\omega)\, \underbrace{p(\vec{n} \mid \omega, \vec{t})}_{\prod_{i=1}^{M} \underbrace{p(n_i \mid \omega, t_i)}_{P[n_i \mid \lambda f(\omega, t_i)]}}$$

prior

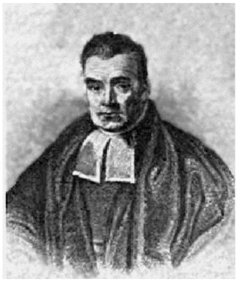
independently estimated

Poisson distribution

fraction of recaptured atoms

arXiv:2204.11816

Release-recapture thermometry: unoptimised times



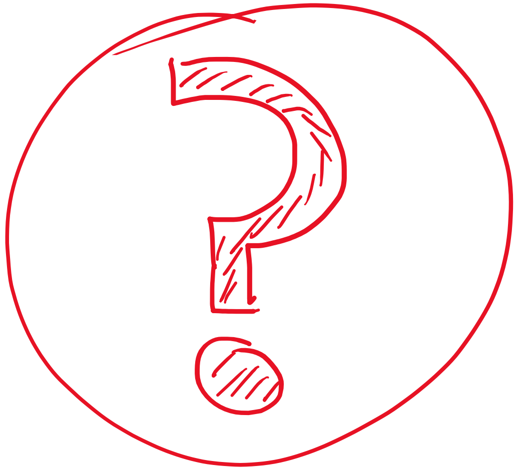
- Physical assumptions:

$$p(\alpha, \vec{n} \mid \vec{t}) = p(\alpha) \prod_{i=1}^{M} P\left[n_i \mid \lambda f(\alpha, t_i)\right]$$

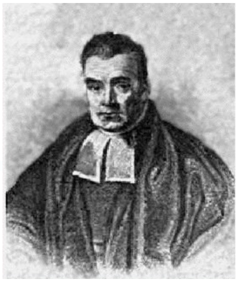
- Missing:

  → prior $p(\alpha)$

  → deviation function $D[\hat{\alpha}(\vec{n}, \vec{t}), \alpha]$



Release-recapture thermometry: unoptimised times



expansion time

chacteristic energy (trap parameters)

fraction of recaptured atoms

$$f(T,t) = \frac{g\{[E_k/(k_B T)]\, W(U_0/E_k)\}}{g[U_0/(k_B T)]}$$

temperature

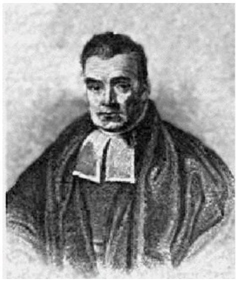
trap depth (energy)

$\begin{cases} g(\cdot) \\ W(\cdot) \equiv \text{Lambert function} \end{cases}$

Invariant under:

$$\begin{cases} T \longrightarrow T' = \gamma T \\ U_0 \longrightarrow U_0' = \gamma U_0 \\ E_k \longrightarrow E_k' = \gamma E_k \end{cases}$$

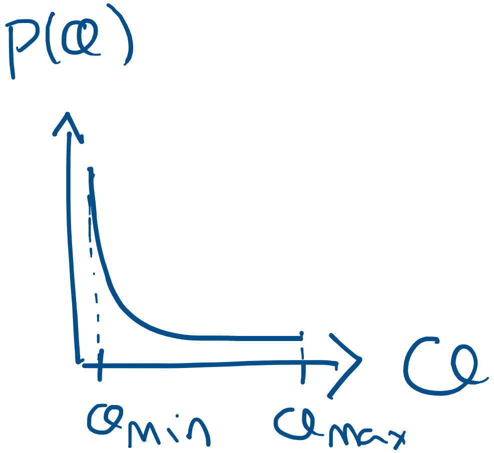
Release-recapture thermometry: unoptimised times



$$\begin{cases} \tau \longrightarrow \tau' = \gamma \tau \\ U_0 \longrightarrow U_0' = \gamma U_0 \\ E_k \longrightarrow E_k' = \gamma E_k \end{cases} \equiv \text{SCALE INVARIANCE}$$

- **Jeffreys's prior**:

$$p(\theta) = \left[\theta \log\left(\frac{\theta_{\max}}{\theta_{\min}}\right)\right]^{-1}$$



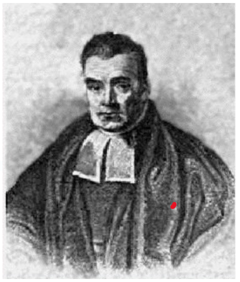
- **Logarithmic error**:

$$\mathcal{D}[\tilde{\theta}(\boldsymbol{n}, \boldsymbol{t}), \theta] = \log^2[\tilde{\theta}(\boldsymbol{n}, \boldsymbol{t})/\theta]$$

IEEE Trans. Syst. Cybern. **4** 227–41 1968

Phys. Rev. Lett. **127**, 190402 (2021)

Release-recapture thermometry: unoptimised times



- **Optimal rule to post-process measurements into a temperature reading:**

$$\tilde{\vartheta}(\boldsymbol{n}, \boldsymbol{t}) = \theta_u \exp\left[\int d\theta\, p(\theta|\boldsymbol{n}, \boldsymbol{t}) \log\left(\frac{\theta}{\theta_u}\right)\right]$$

UNIVERSALLY VALID FOR SCALE ESTIMATION

$$p(\theta|\boldsymbol{n}, \boldsymbol{t}) \propto p(\theta) \prod_{i=1}^{\mu} p(n_i|\theta, t_i)$$

= BAYES THEOREM

Release-recapture thermometry: unoptimised times

How do we report temperature estimates in global quantum thermometry?

$$\tilde{\vartheta}(\boldsymbol{n},\boldsymbol{t}) \pm \Delta\tilde{\vartheta}(\boldsymbol{n},\boldsymbol{t})$$

- Optimal **temperature estimator**:

$$\tilde{\vartheta}(\boldsymbol{n},\boldsymbol{t}) = \theta_u \exp\left[\int d\theta\, p(\theta|\boldsymbol{n},\boldsymbol{t}) \log\left(\frac{\theta}{\theta_u}\right)\right]$$

- **Error bar**:

$$\Delta\tilde{\vartheta}(\boldsymbol{n},\boldsymbol{t}) = \tilde{\vartheta}(\boldsymbol{n},\boldsymbol{t})\sqrt{\bar{\epsilon}_{\mathrm{mle}}(\boldsymbol{n},\boldsymbol{t})}$$

- Measurement-dependent **mean logarithmic error**:

$$\bar{\epsilon}_{\mathrm{mle}}(\boldsymbol{n},\boldsymbol{t}) = \int d\theta\, p(\theta|\boldsymbol{n},\boldsymbol{t}) \log^2\left[\frac{\tilde{\vartheta}(\boldsymbol{n},\boldsymbol{t})}{\theta}\right]$$

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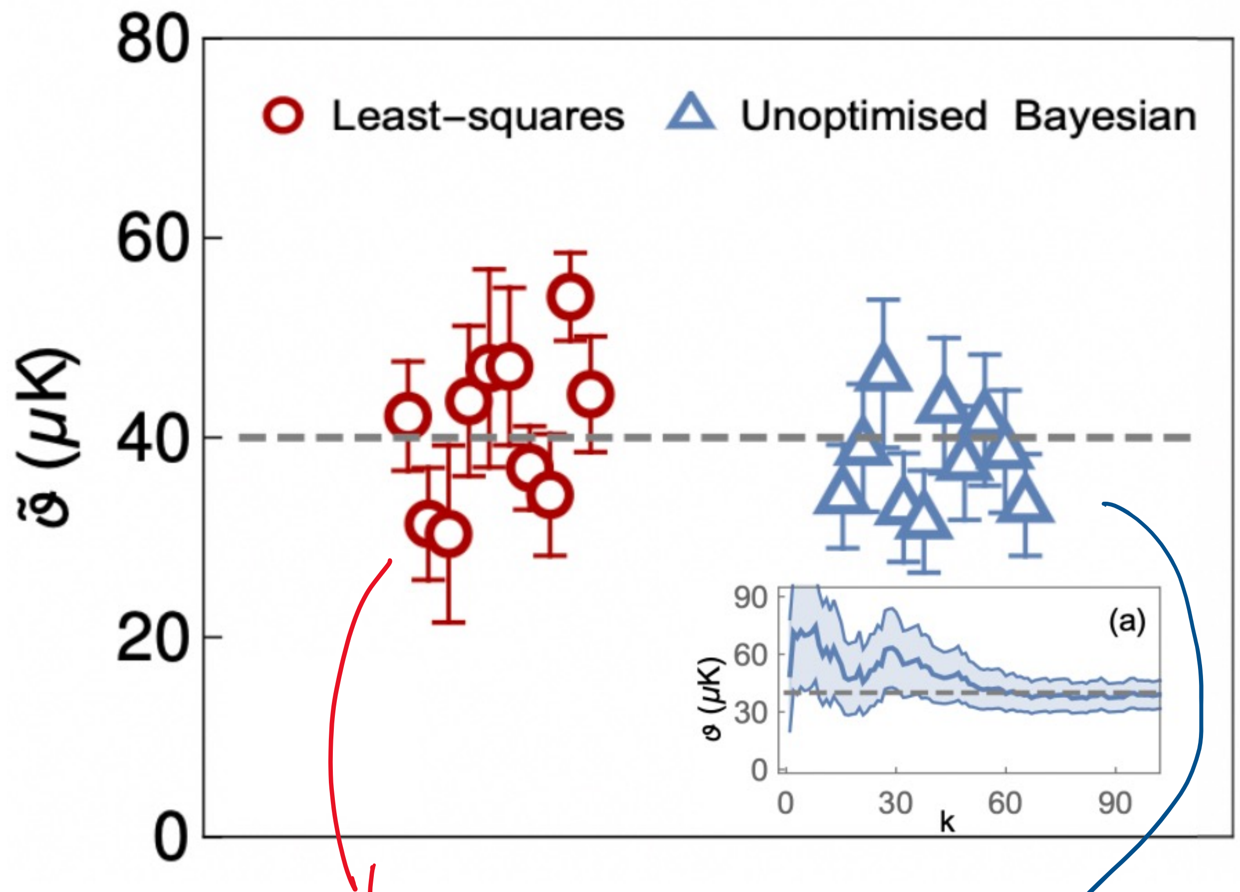
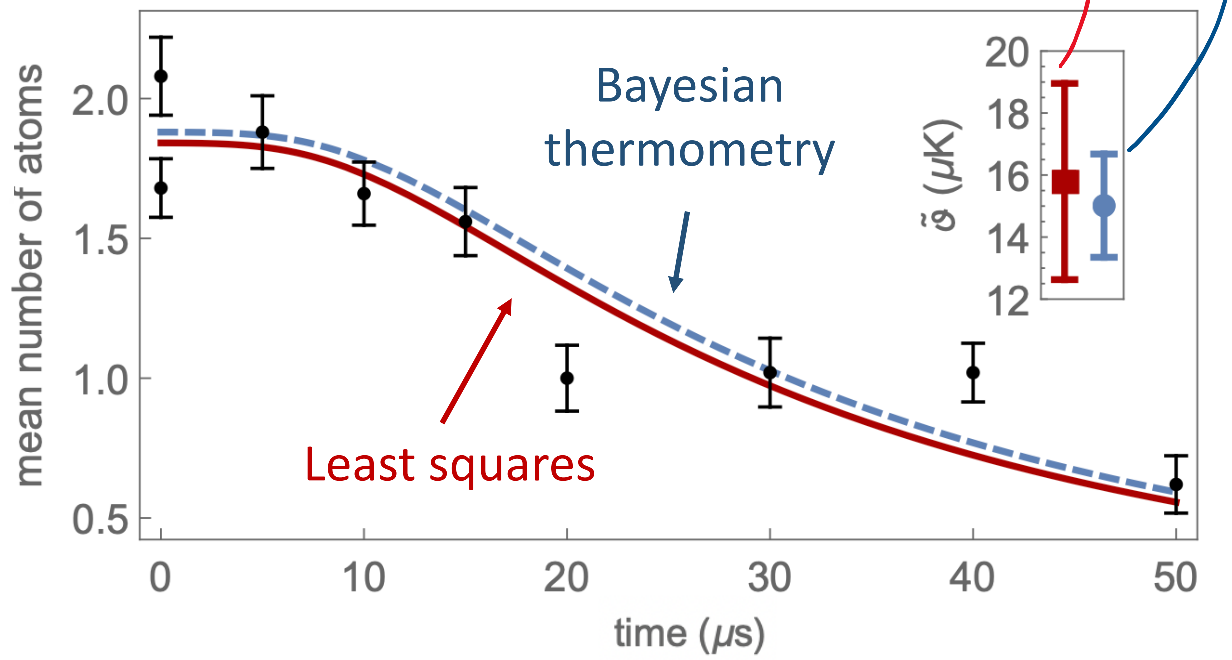


$$\frac{\Delta\tilde{\vartheta}^2}{\tilde{\vartheta}^2} = 0.064$$

$$\frac{\Delta\tilde{\vartheta}^2}{\tilde{\vartheta}^2} = 0.057$$

$$\tilde{\vartheta} = 15.0 \pm 1.7\ \mu K$$

$$\tilde{\vartheta} = 15.8 \pm 3.2\ \mu K$$



III. Maximising information content in an adaptive fashion

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Mean information gain for a single shot (<u>supersedes the Fisher information</u>):

$$\mathcal{K}(t) = \sum\nolimits_n p(n|t) \log^2 \left[\frac{\tilde{\vartheta}(n,t)}{\tilde{\vartheta}_p} \right]$$

information provided by the measurement w.r.t. the optimal prior estimate

where

- optimal *a priori* estimate:

$$\tilde{\vartheta}_p = \theta_u \exp\left[\int d\theta \, p(\theta) \log\left(\frac{\theta}{\theta_u} \right) \right]$$

- evidence:

$$p(n|t) = \int d\theta \, p(\theta) \, p(n|\theta, t)$$

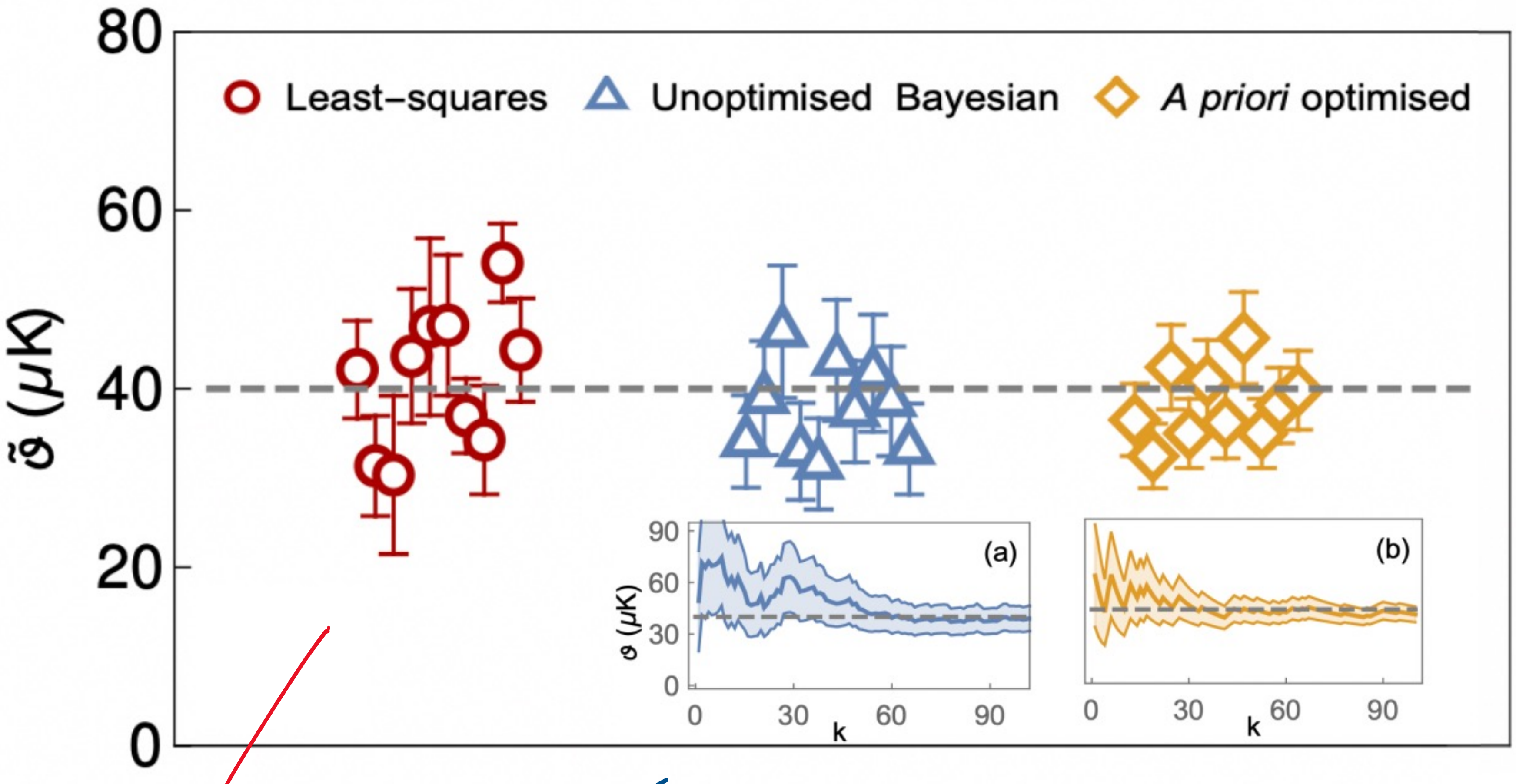
A priori optimised strategy

$$\boldsymbol{t} = (\underbrace{t_s, \ldots, t_s}_{\mu \text{ times}})$$

where t_s is solution to the optimisation problem:

$$\frac{d\kappa(t)}{dt} = 0, \quad \frac{d^2\kappa(t)}{dt^2} < 0.$$

Prescription first proposed in: *New J. Phys.* **21** 043037 2019



$$\frac{\Delta\tilde{\vartheta}^2}{\tilde{\vartheta}^2} = 0.064$$

$$\frac{\Delta\tilde{\vartheta}^2}{\tilde{\vartheta}^2} = 0.057$$

$$\frac{\Delta\tilde{\vartheta}^2}{\tilde{\vartheta}^2} = 0.034$$

$\Rightarrow$ 40 % improvement

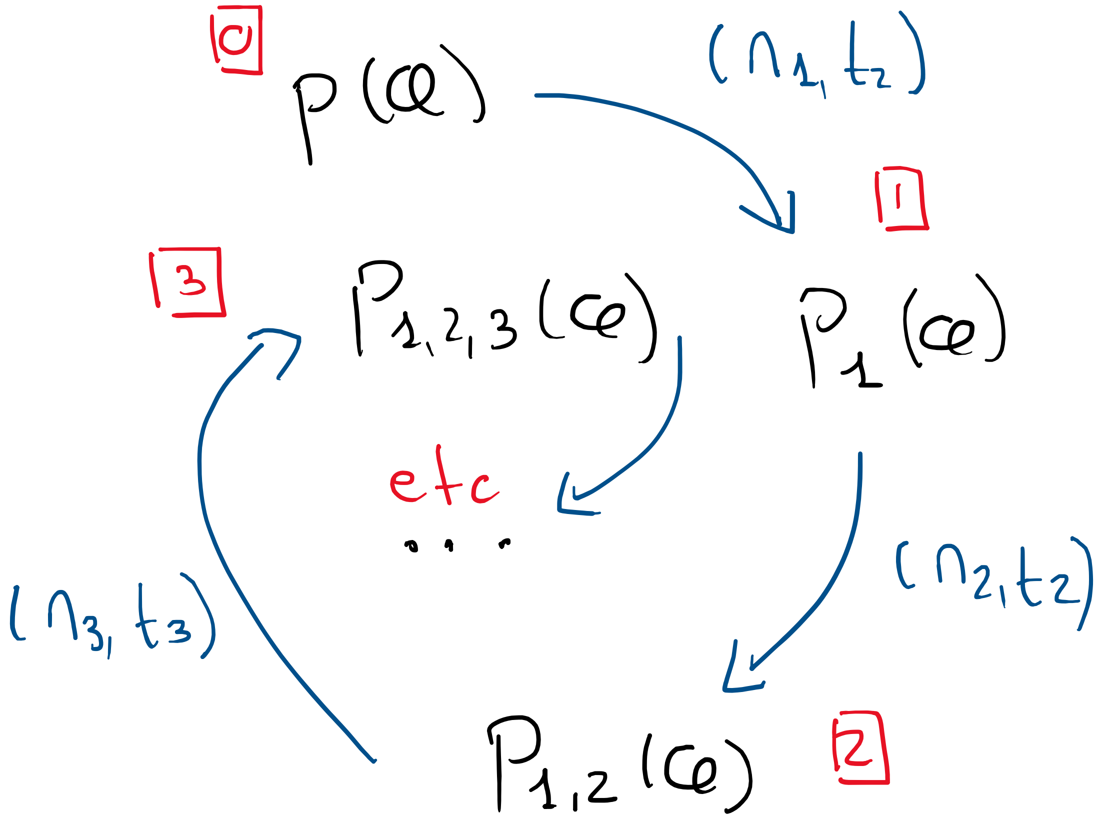
A fully adaptive approach

1. Given the prior $p(\theta)$ and the likelihood $p(n|\theta, t)$ for the first shot, maximise $\mathcal{K}(t)$ over t to find $t_1 = t_s$.

2. Perform a measurement at $t_1 = t_s$ and record n_1.

3. Normalise $p(\theta)\, p(n_1|\theta, t_1)$ and use it as the new 'prior' for a second run [33, 34]. Then apply step 1 to find the optimal expansion time t_2, and measure n_2.

4. Iterate μ times. The resulting data can then be processed using Eqs. (3) and (4).



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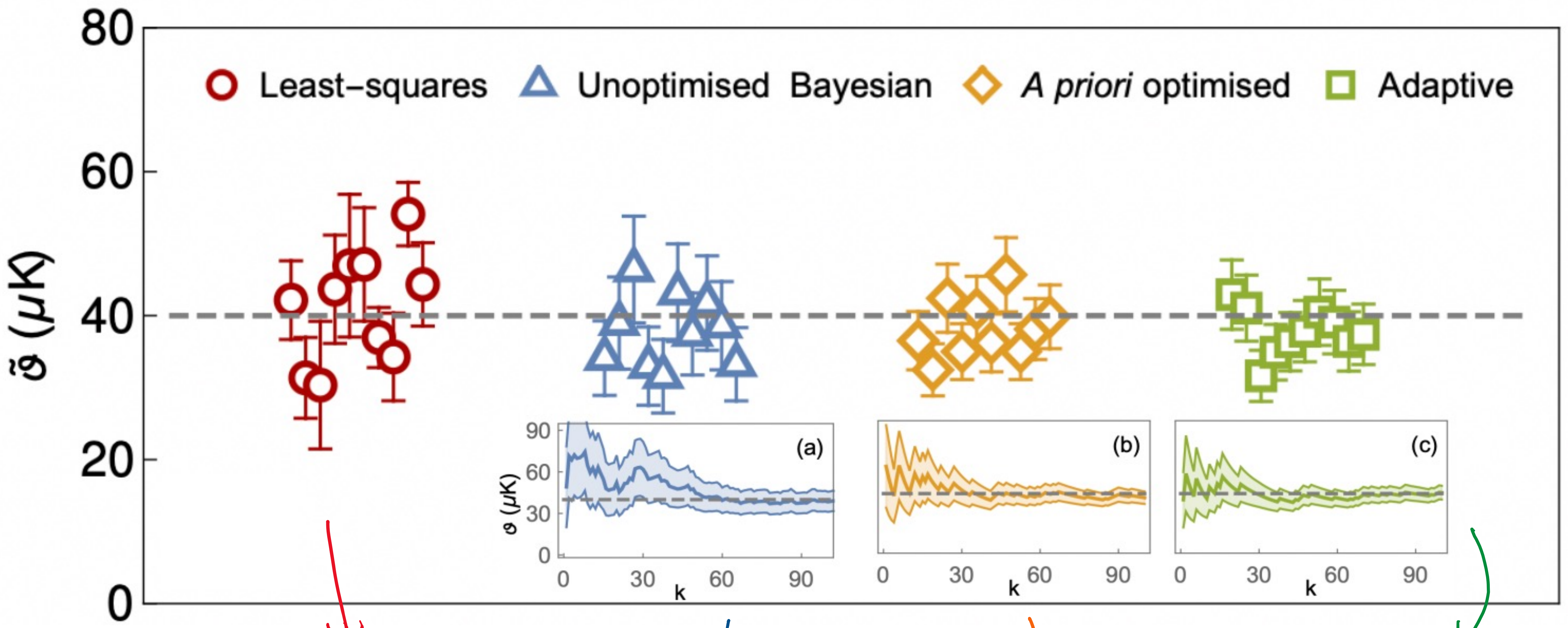
arXiv:2204.11816



$$\frac{\Delta\tilde{\Theta}^2}{\tilde{\Theta}^2} = 0.064$$

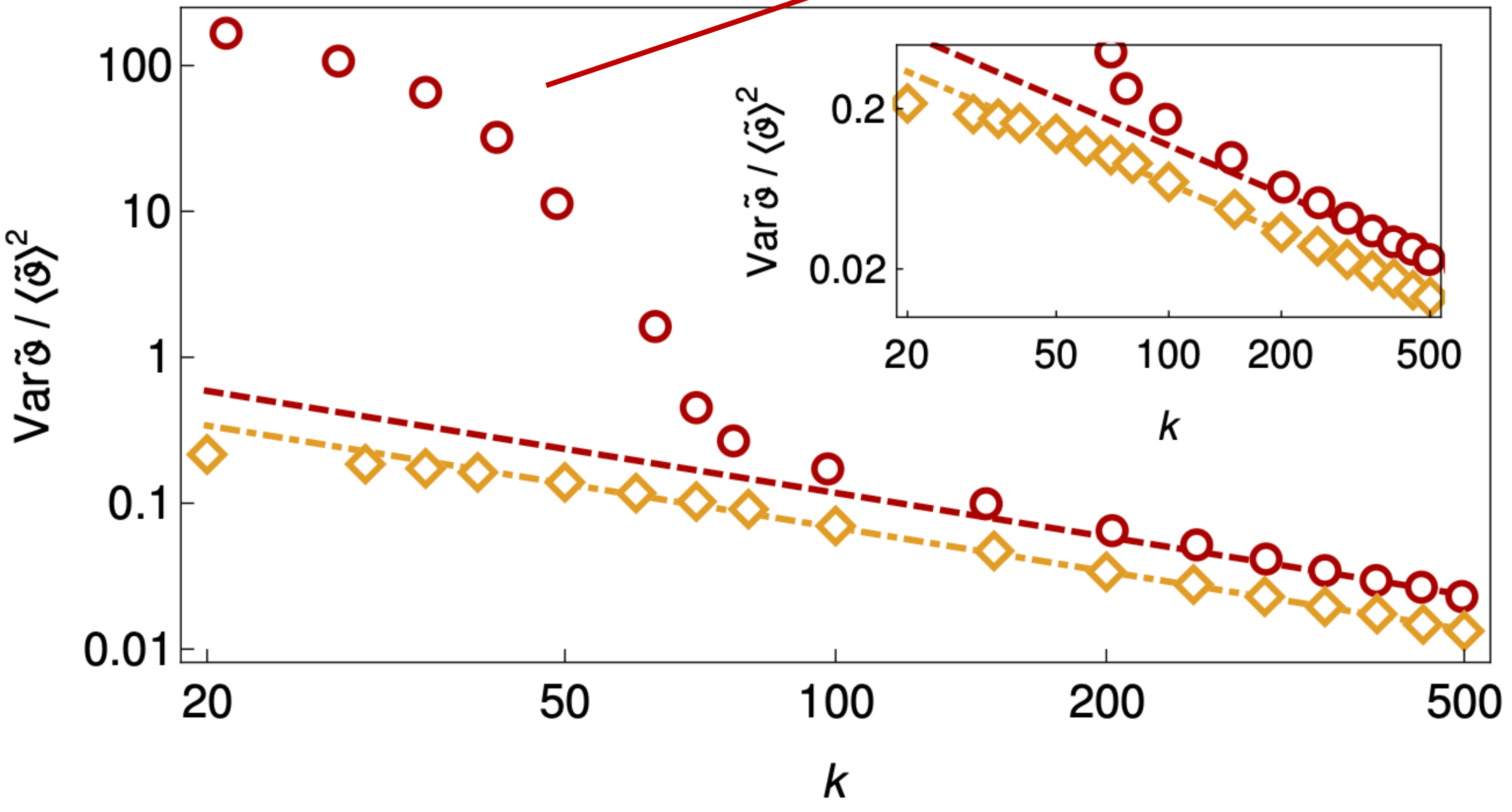
$$\frac{\Delta\tilde{\Theta}^2}{\tilde{\Theta}^2} = 0.057$$

$$\frac{\Delta\tilde{\Theta}^2}{\tilde{\Theta}^2} = 0.034$$

$$\sim \frac{\Delta\tilde{\Theta}^2}{\tilde{\Theta}^2}$$

Precision and convergence

Least squares



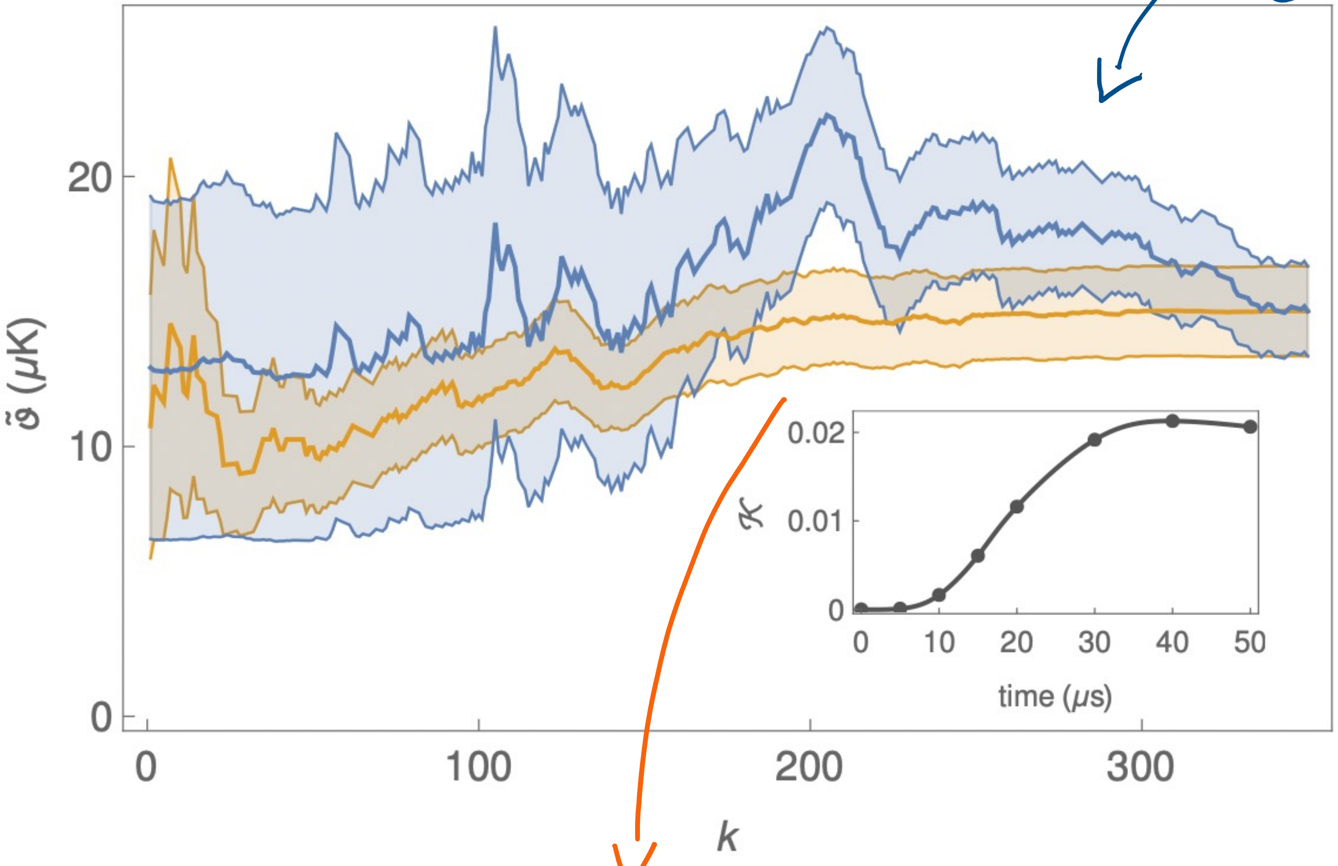
A priori optimised

Twice as much precision with half of the measurement data

arXiv:2204.11816

New J. Phys. **21**, 043037 (2019)

Why does it work?



mimicking the a priori optimised protocol with the available experimental data

reordering events from most to least informative

IV. Conclusions and outlook

Optimal cold atom thermometry using adaptive Bayesian strategies

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(Dated: 24th October 2022)

- *Standard* release-recapture thermometry **is inefficient and wastes resources**.
- **Global quantum thermometry can provide twice as much precision using half of the measurement data**.
- The global-Bayesian framework is applicable to *any* thermometric protocol where temperature plays the role of a scale parameter.
- <u>Next steps</u>: Bayesian formulation of non-equilibrium quantum thermometry under minimal assumptions.

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Quantum Science and Technology

PAPER

Quantum scale estimation

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Global Quantum Thermometry

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(Dated: 30th November 2020)

Thank you for
you attention